Eur. Phys. J. C 28, 353–363 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01180-x

THE EUROPEAN PHYSICAL JOURNAL C

Precision and uncertainties in mass scale predictions in SUSY $SO(10)$ with $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L} \times SU(3)_C$ intermediate breaking

M.K. Parida[a], B. Purkayastha, C.R. Das[b], B.D. Cajee

Physics Department, North-Eastern Hill University, Shillong 793022, India

Received: 9 May 2002 /
Published online: 31 March 2003 –

Abstract. In a class of SUSY $SO(10)$ with $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L} \times SU(3)_C$ ($g_{2\rm L} \neq g_{2\rm R}$) intermediate gauge symmetry, we observe that the prediction on the unification mass ($M_{\rm U}$) is unaffected by Planck-scale-induced gravitational and intermediate-scale threshold effects, although the intermediate scale ($M_{\rm I}$) itself is subject to such corrections. In particular, without invoking the presence of additional lighter scalar degrees of freedom but including plausible and reasonable threshold effects, we find that interesting solutions for neutrino physics corresponding to $M_{\rm I} \simeq 10^{10}$–10^{13} GeV and $M_{\rm U} \simeq (5$–$6) \times 10^{17}$ GeV are permitted in the minimal models. The possibilities of low-mass right-handed gauge bosons corresponding to $M_{\rm I} \simeq 1$–10 TeV consistent with the CERN-LEP data are pointed out in a number of models in which threshold effects are included using effective mass parameters.

1 Introduction

Supersymmetric grand unified theories (GUTs) have been the subject of considerable attention over the past two decades [1–4]. While non-SUSY $SU(5)$ fails to unify the gauge couplings of the standard model, $SU(2)_{\rm L} \times U(1)_Y \times SU(3)_C (\equiv G_{213})$, the SUSY $SU(5)$ and single step breaking of almost all SUSY GUTs exhibit remarkable unification of gauge and Yukawa couplings at $M_{\rm U} \simeq 10^{16}$ GeV consistent with the recent CERN-LEP measurements. Compared to other GUTs, $SO(10)$ has several attractive features. The fermions contained in the spinorial representation $\underline{16} \subset SO(10)$ have just one extra member per generation which is the right-handed neutrino needed to generate light Majorana neutrino masses over a wide range of values through the see-saw mechanism [5]. It explains why there is parity violation at low energies starting from parity conservation at the GUT scale [6,7]. It is the minimal left–right symmetric GUT with natural quark–lepton unification and having $SU(2)_{\rm L} \times SU(2)_{\rm R} \times SU(4)_C$ [7] as its maximal subgroup. It has the potentiality to guarantee R-parity conservation in the Lagrangian.

With $M_{\rm U} \simeq M_N \simeq 10^{16}$ GeV, where M_N is the degenerate right-handed Majorana neutrino mass, the grand desert model through the see-saw mechanism predicts much smaller values of light left-handed Majorana neutrino masses than those needed for understanding neutrinos as a hot dark matter (HDM) candidate along with experimental indications on atmospheric neutrino oscillations and neutrinoless double β decay [8,9], unless substantially lower values of M_N are obtained by a judicious dialing of the Yukawa coupling of the right-handed Majorana neutrino, or via non-renormalizable operators. However, in such cases, one of the most attractive features like b–τ Yukawa unification for smaller values of $\tan\beta$ has to be sacrificed [10,11]. On the other hand, SUSY $SO(10)$ with an intermediate gauge symmetry like $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2213})$ [12–16] or $SU(2)_{\rm L} \times SU(2)_{\rm R} \times SU(4)_C (\equiv G_{224})$ [17–20], while providing a more natural value for M_N, substantially lower than the GUT scale, has the potentialities to account for the b–τ Yukawa unification at the intermediate scale $M_{\rm I} \simeq 10^9$–10^{13} GeV. In this context it has been demonstrated that the desirable values of the G_{2213}-breaking scale with $M_{\rm I} \simeq 10^9$–10^{13} are possible provided that a number of scalar components of full $SO(10)$ Higgs representations are light with masses near the intermediate scale [13,14].

Gravitational corrections due to higher-dimensional operators [21,23] and threshold effects due to superheavy particles have been shown to influence the GUT predictions significantly [17,23–25]. Since neither the superheavy masses contributing to threshold effects near the GUT scale, nor the coefficients of the higher-dimensional operators contributing to gravitational corrections are determined by the grand unified theories, these corrections add to the uncertainties and inaccuracies of the model predictions. In order to remove such limitations of the GUTs, it is important to search for gauge symmetries and

[a] e-mail: mparida@sancharnet.in
[b] e-mail: crdas@email.com

possible representations for which some of the uncertainties could be absent. For example, it has been demonstrated through theorems that in all GUTs with $SU(2)_{\rm L}\times SU(2)_{\rm R}\times SU(4)_C\times P(\equiv G_{224P}, g_{2\rm L}=g_{2\rm R})$ intermediate symmetry, the GUT-threshold and all gravitational corrections on $\sin^2\theta_{\rm W}(M_Z)$ and the intermediate scale are absent [18,19]. The presence of the G_{224P} intermediate gauge symmetry has been found to be essential for these cancellations.

The purpose of this paper is two-fold. For the first time, we demonstrate certain precise results in a class of GUTs with $G_{2213}(g_{2\rm L}\neq g_{2\rm R})$ intermediate gauge symmetry with D-parity broken at the GUT scale [26]. In particular we find that in $SO(10)$ the dominant effect due to the five-dimensional operator is absent on $M_{\rm U}$ leading to the absence of such gravitational corrections on the proton lifetime for $p\to e^+\pi^0$. The threshold effects caused by the spreading of masses around the intermediate scale are also found to be absent on $M_{\rm U}$. Secondly, while exploring uncertainties in the intermediate-scale predictions in SUSY $SO(10)$, we show, for the first time, that the G_{2213} intermediate symmetry is allowed to survive down to $M_{\rm I}\simeq 10^{10}$–10^{13} GeV by threshold and gravitational corrections. We have investigated the impact of threshold effects in SUSY $SO(10)$ models with one pair of $\underline{126}\oplus\overline{\underline{126}}$ and one or two pairs of $\underline{16}\oplus\overline{\underline{16}}$ and find that even $M_{\rm I}\simeq 10$ TeV is allowed, consistent with the CERN-LEP measurements [27], provided that the effective mass parameters at the intermediate or GUT thresholds are a few times heavier or lighter than the corresponding scales.

This paper is organized in the following manner. In Sect. 2 we discuss the analytic formulas for mass scales including threshold and gravitational corrections. In Sect. 3 we derive vanishing corrections due to the five-dimensional operator on the GUT scale and estimate gravitational corrections on the intermediate scale. In Sect. 4 we discuss the threshold effects and their impact on $M_{\rm U}$ and $M_{\rm I}$. The results are summarized with conclusions in Sect. 5.

2 Analytic formulas for mass scales

We consider the following symmetry breaking pattern and derive the analytic formulas for the unification mass $M_{\rm U}$ and the intermediate scale $M_{\rm I}$ including one-loop, two-loop, gravitational and threshold corrections. We have

$$SO(10)\times {\rm SUSY}\xrightarrow[M_{\rm U}]{210} G_{2213}\times{\rm SUSY}$$
$$\xrightarrow[M_{\rm I}]{S} G_{213}\times{\rm SUSY}\xrightarrow[M_Z]{10} U(1)_{em}\times SU(3)_C,$$

where the multiplet S is a component of the $SO(10)$ representations $\underline{16}\oplus\overline{\underline{16}}$ or $\underline{126}\oplus\overline{\underline{126}}$ as the case may be. The renormalization group equations in the presence of the two gauge symmetries G_{213} and G_{2213} below the GUT scale can be written as

$$\frac{1}{\alpha_i(M_Z)}=\frac{1}{\alpha_i(M_{\rm I})}+\frac{a_i}{2\pi}\ln\frac{M_{\rm I}}{M_Z}+\theta_i-\Delta_i,\quad i=1Y,2{\rm L},3C;\tag{1}$$

$$\frac{1}{\alpha_i(M_{\rm I})}=\frac{1}{\alpha_i(M_{\rm U})}+\frac{a'_i}{2\pi}\ln\frac{M_{\rm U}}{M_{\rm I}}+\theta'_i-\Delta'_i-\Delta_i^{\rm NRO},\quad i=2{\rm L},2{\rm R},BL,3C.\tag{2}$$

where the second term in the R.H.S. of (1) and (2) represents one-loop contributions and the third term of both equations are the two-loop terms [28],

$$\theta_i=\frac{1}{4\pi}\sum_j B_{ij}\ln\frac{\alpha_j(M_{\rm I})}{\alpha_j(M_Z)},$$
$$\theta'_i=\frac{1}{4\pi}\sum_j B'_{ij}\ln\frac{\alpha_j(M_{\rm U})}{\alpha_j(M_{\rm I})},\tag{3}$$

$$B_{ij}=\frac{b_{ij}}{a_j},\ B'_{ij}=\frac{b'_{ij}}{a'_j}.\tag{4}$$

While the functions Δ_i include threshold effects at M_Z and $M_{\rm I}$,

$$\Delta_i=\Delta_i^{(Z)}+\Delta_i^{(I)},$$

the Δ'_i include threshold effects at $M_{\rm U}$. The expressions for Δ_i and Δ'_i are given in Sect. 5. The term $\Delta_i^{\rm NRO}$ in (2) contains higher-dimensional-operator effects which modify the boundary condition at $\mu=M_{\rm U}$ as follows:

$$\begin{aligned}\alpha_{2\rm L}(M_{\rm U})(1+\epsilon_{2\rm L})&=\alpha_{2\rm R}(M_{\rm U})(1+\epsilon_{2\rm R})\\&=\alpha_{BL}(M_{\rm U})(1+\epsilon_{BL})\\&=\alpha_{3C}(M_{\rm U})(1+\epsilon_{3C})\\&=\alpha_{\rm G},\end{aligned}\tag{5}$$

leading to

$$\Delta_i^{\rm NRO}=-\frac{\epsilon_i}{\alpha_{\rm G}},\ i=2{\rm L},2{\rm R},BL,3C,\tag{6}$$

where $\alpha_{\rm G}$ is the GUT fine-structure constant. Considering the boundary condition (5) along with (1), (2) and (6) we obtain the following analytic formulas for the mass scales:

$$\ln\frac{M_{\rm I}}{M_Z}=\frac{1}{(AB'-A'B)}\bigg[(AL_S-A'L_\theta)+(A'J_2-AK_2)-\frac{2\pi}{\alpha_{\rm G}}(A\epsilon''-A'\epsilon')+(A'J_\Delta-AK_\Delta)\bigg],\tag{7}$$

$$\ln\frac{M_{\rm U}}{M_Z}=\frac{1}{(AB'-A'B)}\bigg[(B'L_\theta-BL_S)+(BK_2-B'J_2)-\frac{2\pi}{\alpha_{\rm G}}(B'\epsilon'-B\epsilon'')+(BK_\Delta-B'J_\Delta)\bigg],\tag{8}$$

where

$$L_S=\frac{2\pi}{\alpha(M_Z)}\left(1-\frac{8}{3}\frac{\alpha(M_Z)}{\alpha_S(M_Z)}\right),$$
$$L_\theta=\frac{2\pi}{\alpha(M_Z)}\left(1-\frac{8}{3}\sin^2\theta_{\rm W}(M_Z)\right),$$
$$A=a'_{2\rm R}+\frac{2}{3}a'_{BL}-\frac{5}{3}a'_{2\rm L},$$
$$B=\frac{5}{3}(a_Y-a_{2\rm L})-\left(a'_{2\rm R}+\frac{2}{3}a'_{BL}-\frac{5}{3}a'_{2\rm L}\right),$$

$$A' = \left(a'_{2R} + \frac{2}{3}a'_{BL} + a'_{2L} - \frac{8}{3}a'_{3C}\right),$$
$$B' = \frac{5}{3}a_Y + a_{2L} - \frac{8}{3}a_{3C} - \left(a'_{2R} + \frac{2}{3}a'_{BL} + a'_{2L} - \frac{8}{3}a'_{3C}\right),$$
$$J_2 = 2\pi\left[\theta'_{2R} + \frac{2}{3}\theta'_{BL} - \frac{5}{3}\theta'_{2L} + \frac{5}{3}(\theta_Y - \theta_{2L})\right],$$
$$K_2 = 2\pi\left[\theta'_{2R} + \frac{2}{3}\theta'_{BL} + \theta'_{2L} - \frac{8}{3}\theta'_{3C} + \frac{5}{3}\theta_Y + \theta_{2L} - \frac{8}{3}\theta_{3C}\right],$$
$$\epsilon' = \epsilon_{2R} + \frac{2}{3}\epsilon_{BL} - \frac{5}{3}\epsilon_{2L},$$
$$\epsilon'' = \epsilon_{2L} + \epsilon_{2R} + \frac{2}{3}\epsilon_{BL} - \frac{8}{3}\epsilon_{3C},$$
$$J_\Delta = -2\pi\left[\Delta'_{2R} + \frac{2}{3}\Delta'_{BL} - \frac{5}{3}\Delta'_{2L} + \frac{5}{3}(\Delta_Y - \Delta_{2L})\right],$$
$$K_\Delta = -2\pi\left[\Delta'_{2R} + \frac{2}{3}\Delta'_{BL} + \Delta'_{2L} - \frac{8}{3}\Delta'_{3C} + \frac{5}{3}\Delta_Y + \Delta_{2L} - \frac{8}{3}\Delta_{3C}\right]. \quad (9)$$

In the R.H.S. of (7) and (8) the first, second, third and the fourth terms are one-loop, two-loop, gravitational and threshold contributions, respectively. The one-loop and the two-loop beta-function coefficients below the intermediate scale (M_I) are given by [22,28],

$$\begin{pmatrix} a_Y \\ a_{2L} \\ a_{3C} \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix},$$
$$b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}, \; i,j = 1Y, 2L, 3C. \quad (10)$$

Above the intermediate scale, the one-loop and two-loop beta-function coefficients are

$$\begin{pmatrix} a'_{2L} \\ a'_{2R} \\ a'_{BL} \\ a'_{3C} \end{pmatrix} = \begin{pmatrix} n_{10} \\ n_{10} + n_{16} + 4n_{126} \\ 6 + \frac{3}{2}n_{16} + 9n_{126} \\ -3 \end{pmatrix},$$

$$b'_{ij} = \begin{pmatrix} 18 + 7n_{10} & 3n_{10} & 3 & 24 \\ 3n_{10} & 18 + 7n_{10} + 7n_{16} + 48n_{126} & 3 + \frac{3}{2}n_{16} + 24n_{126} & 24 \\ 9 & 9 + \frac{9}{2}n_{16} + 72n_{126} & 7 + \frac{9}{4}n_{16} + 54n_{126} & 8 \\ 9 & 9 & 1 & 14 \end{pmatrix},$$
$$i,j = 2L, 2R, BL, 3C. \quad (11)$$

Including one- and two-loop corrections, we consider a variety of models taking the lighter multiplet to be

$$S = pn_{126} + qn_{16},$$

where p and q are integers. Here $n_{126} = 1$ or $n_{16} = 1$ imply that the components $\Delta_R(1,3,-1,1) + \overline{\Delta}_R(1,3,1,1) \subset \underline{126} \oplus \underline{\overline{126}}$ or $\chi_R(1,2,1/2,1) + \overline{\chi}_R(1,2,-1/2,1) \subset \underline{16} \oplus \underline{\overline{16}}$ of $SO(10)$ have masses close to M_I. Here a minimal model is defined as one with $n_{126} = 1$ or $n_{16} = 1$ where only one set of $\underline{126} \oplus \underline{\overline{126}}$ or $\underline{16} \oplus \underline{\overline{16}}$ is used for G_{2213} breaking. In addition the GUT scale symmetry breaking is carried out by only one representation like $\underline{210}$ or $\underline{45}$ which are needed for decoupling the parity and $SU(2)_R$-breakings. There are non-minimal models in the literature as in [13,14] and in [12], the latter having $n_{16} = 3$. It may be noted that the spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$ gauge symmetry by $\underline{126}$ guarantees automatic conservation of R-parity, whereas the use of $\underline{16}$ instead of $\underline{126}$ leads to R-parity violation. In the latter case it is necessary to impose additional discrete symmetries to maintain the stability of the proton. We use the following input parameters for our analysis [27]:

$$\alpha^{-1}(M_Z) = 128.9 \pm 0.09,\; \alpha_{3C} = 0.119 \pm 0.004,$$
$$\sin^2\theta(M_Z) = 0.23152 \pm 0.00032,\; M_Z = 91.187\,\text{GeV}. \quad (12)$$

Our solutions including only one-loop and two-loop contributions in different models are shown in Table 1. For example, if $n_{16} = 1$ and $n_{126} = 0$ the two-loop values are $M_I = 10^{16.9}$ GeV and $M_U = 10^{16.1}$ GeV. It is clear from Table 1 that up to two-loop level the models do not allow $M_I = 10^{15}$ GeV and in some cases M_I is even greater than M_U which are forbidden. Also it is to be noted that M_U for all models attains a constant value of $10^{16.5}$ GeV. This phenomenon with the occurrence of $M_I \simeq M_U$ has led one to invoke the existence of lighter scalar degrees of freedom in order to bring down the value of the intermediate scale with $M_I \ll M_U$ [13,14].

3 Gravitational corrections on the mass scales

The mechanism of the decoupling of parity and $SU(2)_R$-breakings is implemented in $SO(10)$ by using the Higgs representation $\underline{210}$ or $\underline{45}$ for the symmetry breaking at the GUT scale. Out of these two, the representation $\underline{45}$ does not contribute to the gravitational corrections through the five-dimensional operator since $\mathrm{Tr}(F_{\mu\nu}\Phi_{(45)}F^{\mu\nu})$ vanishes identically. Thus, confining oureselves to the minimal model and using $\underline{210}$ for the $SO(10)$ symmetry breaking at the GUT scale, we demonstrate in this section how the prediction on the unification mass has a vanishing correction due to the five-dimensional operator. We also show how the gravitational effect lowers the intermediate scale by at most two orders of magnitude from the SUSY GUT scale.

Table 1. Mass scales and coefficients for different SUSY $SO(10)$ models including one-loop and two-loop contributions. Also shown are the values of $M_{\rm I}$ including the five-dimensional operator effect while $M_{\rm U}$ remains unaffected

n_{16}	n_{126}	A	B	A'	B'	One-loop $M_{\rm I}$ (GeV)	One-loop $M_{\rm U}$ (GeV)	Two-loop $M_{\rm I}$ (GeV)	Two-loop $M_{\rm U}$ (GeV)	Five-dimensional operator η	$M_{\rm I}$ (GeV)
1	0	$\frac{16}{3}$	4	16	4	$10^{15.96}$	$10^{16.5}$	$10^{16.9}$	$10^{16.11}$	8	$10^{15.9}$
0	1	$\frac{40}{3}$	−4	24	−4	$10^{17.00}$	$10^{16.5}$	$10^{15.2}$	$10^{16.11}$	−8	$10^{14.3}$
2	0	$\frac{22}{3}$	2	18	2	$10^{15.44}$	$10^{16.5}$	$10^{17.6}$	$10^{16.12}$	8	$10^{15.8}$
0	2	$\frac{70}{3}$	−14	34	−14	$10^{16.68}$	$10^{16.5}$	$10^{15.92}$	$10^{16.11}$	−8	$10^{15.6}$
1	1	$\frac{46}{3}$	−6	26	−6	$10^{16.83}$	$10^{16.5}$	$10^{15.5}$	$10^{16.09}$	−8	$10^{14.9}$
2	1	$\frac{52}{3}$	−8	28	−8	$10^{16.74}$	$10^{16.5}$	$10^{15.69}$	$10^{16.12}$	−8	$10^{15.2}$
1	2	$\frac{76}{3}$	−16	36	−16	$10^{16.61}$	$10^{16.5}$	$10^{15.9}$	$10^{16.12}$	−8	$10^{15.6}$
0	3	$\frac{100}{3}$	−24	44	−24	$10^{16.57}$	$10^{16.5}$	$10^{16.9}$	$10^{15.90}$	−8	$10^{15.8}$
3	0	$\frac{28}{3}$	0	20	0	–	$10^{16.5}$	–	–	–	–

3.1 Vanishing gravitational corrections on the unification scale

The super-Higgs representation 210 contains the singlet $\xi(1,1,1)$ under $SU(2)_{\rm L} \times SU(2)_{\rm R} \times SU(4)_C$ which has been noted to be odd under D symmetry that acts like the left–right discrete symmetry ($\equiv$ Parity) [26]. But the neutral component in $\chi(1,1,15)$ of 210 is even under the same D symmetry. $SO(10)$ can be broken to G_{2213} without left–right discrete symmetry by assigning the vacuum expectation value $\langle\xi(1,1,1)\rangle = \langle\chi^0(1,1,15)\rangle \simeq M_{\rm U}$. In this case it has been shown in [25] that the non-renormalizable Lagrangian containing the five-dimensional operator

$$-\frac{\eta}{2M_{Pl}}{\rm Tr}\left(F_{\mu\nu}\Phi_{210}F^{\mu\nu}\right) \tag{13}$$

yields, via (5) and (9),

$$\begin{aligned}\epsilon_{2\rm R} &= -\epsilon_{2\rm L} = -\epsilon_{3C} = \frac{1}{2}\epsilon_{BL} = \epsilon,\\ \epsilon &= \frac{\eta}{16}\frac{M_{\rm U}}{M_{Pl}}\left[\frac{3}{2\pi\alpha_{\rm G}}\right]^{\frac{1}{2}},\\ \epsilon'' &= \epsilon_{2\rm L} + \epsilon_{2\rm R} + \frac{2}{3}\epsilon_{BL} - \frac{8}{3}\epsilon_{3C} = 4\epsilon,\\ \epsilon' &= \epsilon_{2\rm R} + \frac{2}{3}\epsilon_{BL} - \frac{5}{3}\epsilon_{2\rm L} = 4\epsilon,\end{aligned} \tag{14}$$

where $\alpha_{\rm G} = \frac{1}{24.3}$. It is important to note that $\epsilon' = \epsilon''$ identically which has a strong bearing on the prediction of the GUT scale. From (7) and (8) we have the gravitational corrections due to the five-dimensional operator,

$$\left(\ln\frac{M_{\rm I}}{M_Z}\right)_{\rm NRO} = \frac{2\pi(A'\epsilon' - A\epsilon'')}{\alpha_{\rm G}(AB' - A'B)}, \tag{15}$$

$$\left(\ln\frac{M_{\rm U}}{M_Z}\right)_{\rm NRO} = \frac{2\pi(B\epsilon'' - B'\epsilon')}{\alpha_{\rm G}(AB' - A'B)}. \tag{16}$$

Now we demonstrate the vanishing gravitational corrections to the unification mass in the following manner. In all models with decoupled parity and $SU(2)_{\rm R}$-breakings where there are no additional $SU(2)_{\rm L}$- or $SU(3)_C$-multiplets below the GUT scale, except the SM-Higgs doublets near M_Z and $\Delta_{\rm R} \oplus \overline{\Delta}_{\rm R}$ or $\chi_{\rm R} \oplus \overline{\chi}_{\rm R}$ near $M_{\rm I}$ as the case may be,

$$\begin{aligned}a_{2\rm L} &= a'_{2\rm L},\\ a_{3C} &= a'_{3C}.\end{aligned} \tag{17}$$

Using (17) in (9), we obtain

$$\begin{aligned}B = B' &= \frac{5}{3}a_Y - \frac{2}{3}a'_{BL} - a'_{2\rm R},\\ A &= a'_{2\rm R} + \frac{2}{3}a'_{BL} - \frac{5}{3}a'_{2\rm L},\\ A' &= a'_{2\rm R} + \frac{2}{3}a'_{BL} + a'_{2\rm L} - \frac{8}{3}a'_{3C}.\end{aligned} \tag{18}$$

Equations (15) and (16) then yield, with the help of (14) and (18),

$$\begin{aligned}\left(\ln\frac{M_{\rm I}}{M_Z}\right) &= \frac{2\pi(A'\epsilon' - A\epsilon'')}{\alpha_{\rm G}B(A - A')} = -\frac{8\pi\epsilon}{B\alpha_{\rm G}},\\ \left(\ln\frac{M_{\rm U}}{M_Z}\right) &= \frac{2\pi(\epsilon'' - \epsilon')}{\alpha_{\rm G}(A - A')} = 0.\end{aligned} \tag{19}$$

The results given in (19) are valid both in SUSY and non-SUSY GUTs like $SO(10)$, $SO(18)$, and E_6 etc. as long as $\epsilon' = \epsilon''$ as in (14). This suggests an important aspect of the model: that in $SO(10)$ with D-parity broken at the GUT scale, the GUT scale and the proton lifetime are unaffected due to gravitational corrections through the five-dimensional operator.

3.2 Gravitational correction on the intermediate scale

We have examined the effect of the gravitational correction on the intermediate scale originating from the five-dimensional operator given in (13) in different models characterized by (n_{16}, n_{126}) = (0,1), (1,0), (1,1), (2,0), (0,2), (2,1), (1,2), (0,3) and (3,0). The one-loop coefficient A, A', B, B' and numerical results obtained in different cases are shown in Table 1. By varying the parameter η within the range -10 to $+10$, we obtain an intermediate scale $M_{\rm I}$ between 10^{14} to 10^{16} GeV. The maximal effects on $M_{\rm I}$ are found to occur in the minimal model with the $\underline{210}$, $\underline{126} \oplus \overline{126}$ and $\underline{10}$ representations where R-parity is automatically conserved, and we obtain the lowest possible value to be $M_{\rm I} \simeq 10^{14}$ GeV.

4 Threshold effects

So far we have noted that the impact of gravitational corrections on the intermediate scale $M_{\rm I}$ could bring it down to 10^{14} GeV whereas the unification scale remains unaffected. The possibility of $M_{\rm I} \simeq 10^{10}$–10^{12} GeV with b–τ Yukawa unification at $M_{\rm I}$ has been addressed in [13, 14] but with a number of additional Higgs scalars having masses near $M_{\rm I}$, even though they do not contribute to the spontaneous symmetry breaking. But we demonstrate here that when threshold effects are taken into account, the scale $M_{\rm I}$ fits into the desired range of values even if gravitational corrections are ignored and there are no additional scalar degrees of freedom (and superpartners) near the intermediate scale. We also note vanishing corrections on the unification mass ($M_{\rm U}$) due to intermediate-scale threshold effects.

From the analytic formulas, the threshold corrections for the mass scales are

$$\Delta \ln \frac{M_{\rm I}}{M_Z} = \frac{(A'J_\Delta - AK_\Delta)}{(AB' - A'B)}, \tag{20}$$

$$\Delta \ln \frac{M_{\rm U}}{M_Z} = \frac{(BK_\Delta - B'J_\Delta)}{(AB' - A'B)}. \tag{21}$$

We assume the extended survival hypothesis to operate with the consequence that all scalar components of an $SO(10)$ representation which do not contribute to spontaneous symmetry breaking are superheavy. Only lighter degrees of freedom are those G_{2213}-components in $126 \oplus \overline{126}$ or $16 \oplus \overline{16}$ which contribute to spontaneous symmetry breaking at $M_{\rm I}$. Similarly the lightest scalar components with masses near M_Z are up and down type doublets originating from $\underline{10} \subset SO(10)$. The coloured triplets in $\underline{10}$ have masses near the GUT scale. We compute threshold effects on $M_{\rm I}$ and $M_{\rm U}$ using two different methods which have been adopted in the current literature.

Now we refer to Sect. 4.1. Effective mass parameters and effective SUSY threshold have been introduced by Carena, Pokorski and Wagner [29] which have also been exploited in studying threshold effects in minimal SUSY GUTs [22]. Similarly SUSY $SU(5)$ GUT-threshold effects have also been investigated by Langacker and Polonsky [22] by introducing another set of effective mass parameters near the GUT scale. For the present analysis we utilize the same set of effective mass parameters at the SUSY scale as in [22] but use two new sets of effective mass parameters at $M_{\rm I}$ and $M_{\rm U}$. Although the effective mass parameters corresponding to the SUSY threshold has been determined approximately using experimental measurements or well-known estimations of the actual masses, such determinations for the effective mass parameters at higher thresholds have not been carried out due to lack of experimental data or adequate estimations of the superheavy masses. In view of this we adopt a procedure similar to that outlined in [22] and assume these effective mass parameters to be a few times heavier or lighter than the corresponding mass scales.

Table 2. The heavy Higgs content of the $SO(10)$ model with G_{2213} intermediate symmetry. The G_{213} submultiplets acquire masses close to $M_{\rm I}$ when G_{2213} is broken

$SO(10)$ representation	G_{213} multiplet	$b'_{2\rm L}, b'_{1Y}, b'_{3C}$
$\underline{16}$	$(1,0,1)$	$(0,0,0)$
$\overline{16}$	$(1,-1,1)$	$(0,\frac{3}{5},0)$
$\underline{16}$	$(1,0,1)$	$(0,0,0)$
$\overline{16}$	$(1,-1,1)$	$(0,\frac{3}{5},0)$
$\underline{16}$ [$\overline{16}$]	$(1,0,1)$	$(0,0,0)$
	$(1,1,1)[(1,-1,1)]$	$(0,\frac{3}{5},0)$
$\underline{126}$	$(1,2,1)$	$(0,\frac{12}{5},0)$
	$(1,0,1)$	$(0,0,0)$
$\overline{126}$	$(1,-1,1)$	$(0,\frac{3}{5},0)$
	$(1,-2,1)$	$(0,\frac{12}{5},0)$

Next we refer to Sect. 4.2. Without introducing effective mass parameters, threshold effects have also been computed conventionally by assigning specific and plausible values of masses to the superheavy scalar components in non-SUSY GUTs as well as SUSY theories [30–34]. This method will be adopted below in a separate analysis. Following a result due to Shifman, masses used for the estimation of threshold effects have been assumed to be bare masses as the wave function renormalization has been shown to get cancelled by two-loop effects [35].

In both these cases we find interesting solutions even when the masses are assigned their expected values and are taken to be a few times heavier or lighter than the corresponding scales.

4.1 Threshold effects with effective mass parameters

Including threshold corrections, we have investigated three models corresponding to (n_{16}, n_{126}) = (1, 0), (2, 0), (0, 1) with $\underline{45} \oplus \underline{54}$, for $SO(10)$ breaking and the other three models corresponding to (n_{16}, n_{126}) = (1, 0), (2, 0), (0, 1) with $\underline{210}$. The superheavy components in these models having masses near $M_{\rm I}$ and $M_{\rm U}$ are shown in Tables 2 and 3, respectively. It is clear that threshold effects on $M_{\rm U}$ and $M_{\rm I}$ can be estimated once $M'_i(i = 1Y, 2{\rm L}, 3C)$ and $M''_i(i = 2{\rm L}, 2{\rm R}, BL, 3C)$ as defined in (22)–(24) below

Table 3. Same as Table 2, but here the G_{2213} submultiplets acquire masses close to $M_{\rm U}$ when $SO(10)$ is broken

$SO(10)$ representation	G_{2213} submultiplet	$b''_{2L}, b''_{2R}, b''_{BL}, b''_{3C}$
$\underline{210}$	$(2,2,\pm\frac{1}{3},6)$	$(6,6,4,10)$
	$(2,2,\pm\frac{1}{3},3)$	$(3,3,2,2)$
	$(2,2,\pm1,1)$	$(1,1,6,0)$
	$(3,1,\pm\frac{2}{3},3)$	$(6,0,6,\frac{3}{2})$
	$(1,3,\pm\frac{2}{3},3)$	$(0,6,6,\frac{3}{2})$
	$(3,1,0,8)$	$(16,0,0,9)$
	$(1,3,0,8)$	$(0,16,0,9)$
	$(3,1,0,1)$	$(2,0,0,0)$
	$(1,3,0,1)$	$(0,2,0,0)$
	$(1,1,0,8)$	$(0,0,0,3)$
$\underline{45}$	$(3,1,0,1)$	$(2,0,0,0)$
	$(1,3,0,1)$	$(0,2,0,0)$
	$(1,1,0,8)$	$(0,0,0,3)$
$\underline{54}$	$(3,3,0,1)$	$(6,6,0,0)$
	$(1,1,\pm\frac{2}{3},6)$	$(0,0,4,\frac{5}{2})$
	$(1,1,0,8)$	$(0,0,0,3)$
	$(2,2,\pm\frac{1}{3},3)$	$(3,3,2,2)$
$\underline{16}\ [\overline{\underline{16}}]$	$(2,1,-\frac{1}{2},1)\ [(2,1,\frac{1}{2},1)]$	$(\frac{1}{2},0,\frac{3}{4},0)$
	$(2,1,\frac{1}{6},3)\ [(2,1,-\frac{1}{6},\overline{3})]$	$(\frac{3}{2},0,\frac{1}{4},1)$
	$(1,2,-\frac{1}{6},3)\ [(1,2,\frac{1}{6},\overline{3})]$	$(0,\frac{3}{2},\frac{1}{4},1)$
$\underline{126}\ [\overline{\underline{126}}]$	$(1,3,-\frac{1}{3},\overline{6})\ [(1,3,\frac{1}{3},6)]$	$(0,12,3,\frac{15}{2})$
	$(1,3,\frac{1}{3},\overline{3})\ [(1,3,-\frac{1}{3},3)]$	$(0,6,\frac{3}{2},\frac{3}{2})$
	$(3,1,\frac{1}{3},6)\ [(3,1,-\frac{1}{3},\overline{6})]$	$(12,0,3,\frac{15}{2})$
	$(3,1,-\frac{1}{3},3)\ [(3,1,\frac{1}{3},\overline{3})]$	$(6,0,\frac{3}{2},\frac{3}{2})$
	$(3,1,-1,1)[(3,1,1,1)]$	$(2,0,\frac{9}{2},0)$
	$(2,2,\frac{2}{3},3)\ [(2,2,1,-\frac{2}{3},\overline{3})]$	$(3,3,8,2)$
	$(2,2,-\frac{2}{3},\overline{3})\ [(2,2,\frac{2}{3},3)]$	$(3,3,8,2)$
	$(2,2,0,8)[(2,2,0,8)]$	$(8,8,0,12)$
	$(2,2,0,1)[(2,2,0,1)]$	$(1,1,0,0)$
	$(1,1,-\frac{1}{3},3)\ [(1,1,\frac{1}{3},\overline{3})]$	$(0,0,\frac{1}{2},\frac{1}{2})$
	$(1,1,\frac{1}{3},\overline{3})\ [(1,1,-\frac{1}{3},3)]$	$(0,0,\frac{1}{2},\frac{1}{2})$
$\underline{10}$	$(1,1,\pm\frac{1}{3},3)$	$(0,0,\frac{1}{2},\frac{1}{2})$

are known. In any model the superheavy masses near any particular symmetry breaking scale can be parametrized in terms of the corresponding effective mass parameters [22,29]. In the present model there are three such relations corresponding to the three symmetry breaking scales i.e., $\mu = M_{\rm SUSY} = M_Z$, $\mu = M_{\rm I}$ and $\mu = M_{\rm U}$,

$$\Delta_i^Z = \sum_\alpha \frac{b_i^\alpha}{2\pi}\ln\frac{M_\alpha}{M_Z} = \frac{b_i}{2\pi}\ln\frac{M_i}{M_Z},$$
$$i = 1Y, 2{\rm L}, 3C;\ \mu = M_Z; \qquad (22)$$

$$\Delta_i^I = \sum_\alpha \frac{b_i'^\alpha}{2\pi}\ln\frac{M'_\alpha}{M_{\rm I}} = \frac{b'_i}{2\pi}\ln\frac{M'_i}{M_{\rm I}},$$
$$i = 1Y, 2{\rm L}, 3C;\ \mu = M_{\rm I}; \qquad (23)$$
$$\Delta_i^U = \sum_\alpha \frac{b_i''^\alpha}{2\pi}\ln\frac{M''_\alpha}{M_{\rm U}} = \frac{b''_i}{2\pi}\ln\frac{M''_i}{M_{\rm U}},$$
$$i = 2{\rm L}, 2{\rm R}, BL, 3C;\ \mu = M_{\rm U}; \qquad (24)$$

where α refers to the actual G_{213} submultiplet near $\mu = M_Z$, $M_{\rm I}$ or the G_{2213} submultiplet near $\mu = M_{\rm U}$, and M_α, M'_α or M''_α refer to the actual component masses. The coefficients $b'_i = \sum b_i'^{(\alpha)}$ and $b''_i = \sum b_i''^{(\alpha)}$ have been defined in (22)–(24) following [22,29]. The numbers b_i^α refer to the one-loop coefficients of the multiplet α under the gauge subgroup $U(1)_Y$, $SU(2)_{\rm L}$, $SU(2)_{\rm R}$, $SU(3)_C$, $U(1)_{B-L}$ etc. Using (9) and (20)–(24) we have obtained contributions to threshold effects on the two mass scales, $\Delta\ln\frac{M_{\rm I}}{M_Z}$ and $\Delta\ln\frac{M_{\rm U}}{M_Z}$, as shown in Table 4 in terms of the effective mass parameters $M'_i(i = 1Y, 2{\rm L}, 3C)$ and $M''_i(i = 2{\rm L}, 2{\rm R}, BL, 3C)$. The numerical entries in Table 4 denote threshold effects at M_Z estimated using the effective mass parameters of [22].

Using one-loop coefficients from Tables 2 and 3 and the effective mass parameters denoted as primes at the intermediate scale and as double primes at the GUT scale, the analytic expressions for the threshold corrections are presented in Table 4 where different models have also been defined. A remarkable feature is that $\Delta\ln\frac{M_{\rm U}}{M_Z}$ has vanishing corrections due to intermediate-scale threshold effects as the corresponding expressions contain no term involving any of the parameters like $M'_{2{\rm L}}$, $M'_{2{\rm R}}$, M'_{BL} or M'_{3C}. Further, the effective mass parameters $M''_{2{\rm R}}$ and M''_{BL} have vanishing contributions to the threshold effects on the unification mass. Another notable feature is that corrections due to $M'_{2{\rm L}}$ and M'_{3C} are absent in $\Delta\ln\frac{M_{\rm I}}{M_Z}$. There is only a small correction due to M'_{1Y}.

Equation (22) has been utilized in [22] to compute only one set of values of M_1, M_2, M_3 in MSSM from the model predictions on M_α. But, since such predictions are also model dependent, several other assumed values of the effective mass parameters have been utilized for the computation. At present no experimental or theoretical information is available on the actual values of the superheavy masses around $M_{\rm I}$ and $M_{\rm U}$, although theoretically it is natural to assume these masses to spread around the corresponding scales by a factor bounded by $\frac{1}{10}$ and 10. In the present case, in the absence of actual values of component masses in the model, we make quite plausible and reasonable assumptions on M'_i and M''_i for the computation. In our analysis the effective mass parameters M'_i or M''_i are taken to vary between $\frac{1}{5}$–5 times the relevant scale of symmetry breaking, i.e., $M_{\rm I}$ or $M_{\rm U}$. Numerical solutions to different allowed values of the mass scales corresponding to different choices of the effective mass parameters are presented in Table 5.

Certain important features of these solutions are noteworthy. The minimal model I with one set of $\underline{210}, \underline{16}\oplus\overline{\underline{16}}$, and $\underline{10}$ permits an intermediate scale in the interesting range of 10^3–10^{13} GeV for reasonable choices of the ef-

Table 4. Threshold effects on mass scales in different SUSY $SO(10)$ models using effective mass parameters

Model	Representation content	$\Delta \ln \frac{M_\mathrm{I}}{M_Z}$	$\Delta \ln \frac{M_\mathrm{U}}{M_Z}$
I	$\underline{210}, \underline{16} \oplus \overline{\underline{16}}, \underline{10}$	$\frac{53}{4} \ln \frac{M''_{2\mathrm{R}}}{M_\mathrm{U}} + \frac{103}{12} \ln \frac{M''_{BL}}{M_\mathrm{U}} - \frac{81}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} + \frac{56}{3} \ln \frac{M''_{3C}}{M_\mathrm{U}} - 1.33$	$\frac{27}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - 14 \ln \frac{M''_{3C}}{M_\mathrm{U}} + 0.117$
II	$\underline{210}, 2(\underline{16} \oplus \overline{\underline{16}}), \underline{10}$	$28 \ln \frac{M''_{2\mathrm{R}}}{M_\mathrm{U}} + 18 \ln \frac{M''_{BL}}{M_\mathrm{U}} - \frac{203}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} + 55 \ln \frac{M''_{3C}}{M_\mathrm{U}} + \frac{3}{2} \ln \frac{M'_{1Y}}{M_\mathrm{I}} - 2.78$	$\frac{29}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - 15 \ln \frac{M''_{3C}}{M_\mathrm{U}} + 0.122$
III	$\underline{210}, \underline{126} \oplus \overline{\underline{126}}, \underline{10}$	$-29 \ln \frac{M''_{2\mathrm{R}}}{M_\mathrm{U}} - \frac{55}{3} \ln \frac{M''_{BL}}{M_\mathrm{U}} + 150 \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - \frac{305}{3} \ln \frac{M''_{3C}}{M_\mathrm{U}} - \frac{9}{4} \ln \frac{M'_{1Y}}{M_\mathrm{I}} - 1.56$	$30 \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - \frac{121}{4} \ln \frac{M''_{3C}}{M_\mathrm{U}} + 0.105$
IV	$\underline{45}, \underline{54}, \underline{16} \oplus \overline{\underline{16}}, \underline{10}$	$\frac{5}{4} \ln \frac{M''_{2\mathrm{R}}}{M_\mathrm{U}} + \frac{7}{12} \ln \frac{M''_{BL}}{M_\mathrm{U}} - \frac{9}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} + \frac{8}{3} \ln \frac{M''_{3C}}{M_\mathrm{U}} - 1.33$	$\frac{3}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - 2 \ln \frac{M''_{3C}}{M_\mathrm{U}} + 0.117$
V	$\underline{45}, \underline{54}, 2(\underline{16} \oplus \overline{\underline{16}}), \underline{10}$	$4 \ln \frac{M''_{2\mathrm{R}}}{M_\mathrm{U}} + 2 \ln \frac{M''_{BL}}{M_\mathrm{U}} - \frac{35}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} + 11 \ln \frac{M''_{3C}}{M_\mathrm{U}} + \frac{3}{2} \ln \frac{M'_{1Y}}{M_\mathrm{I}} - 2.78$	$\frac{5}{2} \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - 3 \ln \frac{M''_{3C}}{M_\mathrm{U}} + 0.122$
VI	$\underline{45}, \underline{54}, \underline{126} \oplus \overline{\underline{126}}, \underline{10}$	$-17 \ln \frac{M''_{2\mathrm{R}}}{M_\mathrm{U}} - \frac{31}{3} \ln \frac{M''_{BL}}{M_\mathrm{U}} + 90 \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - \frac{185}{3} \ln \frac{M''_{3C}}{M_\mathrm{U}} - \frac{9}{4} \ln \frac{M'_{1Y}}{M_\mathrm{I}} - 1.56$	$18 \ln \frac{M''_{2\mathrm{L}}}{M_\mathrm{U}} - \frac{37}{2} \ln \frac{M''_{3C}}{M_\mathrm{U}} + 0.105$

fective mass parameters having string scale unification $M_\mathrm{U} \simeq (5-6) \times 10^{17}$ GeV. Also the $SO(10)$ model with $\underline{210}, \underline{126} \oplus \overline{\underline{126}}$ and $\underline{10}$ allows $M_\mathrm{I} \simeq 5.3 \times 10^{11}$ GeV with high unification mass close to the string scale, $M_\mathrm{U} \simeq 5.6 \times 10^{17}$ GeV. We also note that the intermediate-scale solution with $M_\mathrm{I} \simeq 10^{11}$–$10^{13}$ GeV is maintained irrespective of the fact whether a $\underline{210}$ or a $\underline{45} \oplus \underline{54}$ or even a $\underline{45} \oplus \underline{210}$ is used for the GUT-scale symmetry breaking.

Although solutions with intermediate scale $M_\mathrm{I} \simeq 10^{11}$–$10^{13}$ GeV are also possible due to threshold effects with superheavy masses as shown in Sect. 4.2, a special and notable feature with effective mass parameters is the possibility of low-mass right-handed gauge bosons corresponding to $M_\mathrm{I} \simeq 1$–10 TeV in all the six models, minimal or non-minimal. These solutions are also indicated in Table 5. Such low-mass right-handed gauge bosons might be tested through experimentally detectable $V + A$ currents in the future [6].

4.2 Threshold effects with superheavy masses

In this subsection, instead of using effective mass parameters, we estimate threshold effects with reasonable choices of the values of the masses of the superheavy components of Higgs scalars and their superpartners in three models corresponding to $(n_{16}, n_{126}) = (1,0), (2,0), (0,1)$ with $\underline{45} \oplus \underline{54}$, and three other models corresponding to $(n_{16}, n_{126}) = (1,0), (2,0), (0,1)$ with $\underline{210}$.

The expression for the threshold effect in terms of actual superheavy component masses M'_α which have values near M_I is given by (23),

$$\Delta_i^I = \sum_\alpha \frac{b_i'^\alpha}{2\pi} \ln \frac{M'_\alpha}{M_\mathrm{I}},\ i = 1Y, 2\mathrm{L}, 3C. \tag{25}$$

The superheavy components α contained in different models which have masses near M_I are shown in Table 2. Similarly, the threshold effect at $\mu = M_\mathrm{U}$ is given by (24),

$$\Delta_i^U = \sum_\alpha \frac{b_i''^\alpha}{2\pi} \ln \frac{M''_\alpha}{M_\mathrm{U}},\ i = 2\mathrm{L}, 2\mathrm{R}, BL, 3C. \tag{26}$$

The superheavy components α contained in different models which have masses near M_U are given in Table 3. While computing threshold effects, we have assumed all the multiplets belonging to an $SO(10)$ representation "H" to have

Table 5. Predictions on mass scales M_I and M_U including the threshold effect with effective mass parameters

Model	Two-loop (GeV)	M'_{1Y}	$M''_{2\mathrm{L}}$	$M''_{2\mathrm{R}}$	M''_{BL}	M''_{3C}	M_I (GeV)	M_U (GeV)
I	$M_\mathrm{I}=10^{16.9}$	–	$4M_\mathrm{U}$	$3M_\mathrm{U}$	$3M_\mathrm{U}$	$2.95M_\mathrm{U}$	1.36×10^{11}	5.2×10^{17}
	$M_\mathrm{U}=10^{16.11}$	–	$5M_\mathrm{U}$	$3.5M_\mathrm{U}$	$3.5M_\mathrm{U}$	M_U	2×10^{11}	1.5×10^{17}
		–	$3M_\mathrm{U}$	M_U	M_U	$2.2M_\mathrm{U}$	1.6×10^{3}	6.5×10^{17}
II	$M_\mathrm{I}=10^{17.6}$	M_I	$3M_\mathrm{U}$	$4M_\mathrm{U}$	$2M_\mathrm{U}$	$2.5M_\mathrm{U}$	1.0×10^{11}	1.3×10^{17}
	$M_\mathrm{U}=10^{16.12}$	M_I	$3M_\mathrm{U}$	$3.3M_\mathrm{U}$	$3.3M_\mathrm{U}$	$2.27M_\mathrm{U}$	2.78×10^{11}	5.58×10^{17}
		$\frac{1}{3}M_\mathrm{I}$	$4M_\mathrm{U}$	$5M_\mathrm{U}$	$2M_\mathrm{U}$	$3M_\mathrm{U}$	1.3×10^{3}	5.55×10^{17}
III	$M_\mathrm{I}=10^{15.2}$	M_I	M_U	M_U	$2M_\mathrm{U}$	M_U	1.11×10^{9}	1.46×10^{16}
	$M_\mathrm{U}=10^{16.11}$	$3.5M_\mathrm{I}$	$2.27M_\mathrm{U}$	$3.5M_\mathrm{U}$	$3.5M_\mathrm{U}$	$2M_\mathrm{U}$	5.3×10^{11}	5.5×10^{17}
		M_I	M_U	M_U	$5M_\mathrm{U}$	M_U	1.2×10^{3}	1.46×10^{16}
IV	$M_\mathrm{I}=10^{16.9}$	–	$2M_\mathrm{U}$	M_U	M_U	M_U	1.84×10^{12}	3.34×10^{17}
	$M_\mathrm{U}=10^{16.11}$	–	$2M_\mathrm{U}$	M_U	M_U	$\frac{1}{1.5}M_\mathrm{U}$	1.23×10^{11}	2.53×10^{18}
		–	$3M_\mathrm{U}$	$\frac{1}{5}M_\mathrm{U}$	$\frac{1}{5}M_\mathrm{U}$	$\frac{1}{2}M_\mathrm{U}$	1.2×10^{3}	6.6×10^{19}
V	$M_\mathrm{I}=10^{17.6}$	M_I	M_U	$\frac{1}{1.5}M_\mathrm{U}$	$5M_\mathrm{U}$	$\frac{1}{2}M_\mathrm{U}$	1.77×10^{12}	9.94×10^{17}
	$M_\mathrm{U}=10^{16.12}$	M_I	M_U	$\frac{1}{1.2}M_\mathrm{U}$	$\frac{1}{5}M_\mathrm{U}$	M_U	2.84×10^{11}	1.48×10^{16}
		$\frac{1}{5}M_\mathrm{I}$	$2M_\mathrm{U}$	M_U	M_U	M_U	6.3×10^{3}	6.7×10^{17}
VI	$M_\mathrm{I}=10^{15.2}$	$5M_\mathrm{I}$	$\frac{1}{2}M_\mathrm{U}$	M_U	$\frac{1}{5}M_\mathrm{U}$	M_U	1.0×10^{14}	6.75×10^{17}
	$M_\mathrm{U}=10^{16.11}$	M_I	M_U	$\frac{1}{1.2}M_\mathrm{U}$	$3M_\mathrm{U}$	M_U	4×10^{11}	1.46×10^{16}
		$5M_\mathrm{I}$	M_U	$\frac{1}{1.1}M_\mathrm{U}$	$5M_\mathrm{U}$	$1.1M_\mathrm{U}$	3.7×10^{3}	1.87×10^{15}

the same degenerate mass M_H. For example, all the superheavy components in $\underline{45}$ given in Table 3 near $\mu = M_\mathrm{U}$ have been subjected to the following degeneracy condition:

$$M''(3,1,0,1) = M''(1,3,0,1) = M''(1,1,0,8) = M_{45}.$$

Similarly for $\underline{210}$, $\underline{54}$, $\underline{16}\oplus\underline{\overline{16}}$, $\underline{126}\oplus\underline{\overline{126}}$ and $\underline{10}$

$$M''\left(2,2,\pm\frac{1}{3},6\right) = M''\left(2,2,\pm\frac{1}{3},3\right) = ... = M_{210},$$

$$M''(3,3,0,1) = M''\left(1,1,\pm\frac{2}{3},6\right) = ... = M_{54},$$

$$M''\left(2,1,-\frac{1}{2},1\right) = M''\left(2,1,\frac{1}{6},3\right) = ... = M_{16},$$

$$M''\left(1,3,-\frac{1}{3},\overline{6}\right) = M''\left(1,3,\frac{1}{3},\overline{3}\right) = ... = M_{126},$$

$$M''\left(1,1,\pm\frac{1}{3},3\right) = M_{10}.$$

All the heavy masses near $\mu = M_\mathrm{I}$ are assumed to have the same mass M'. We have obtained contributions to threshold effects on the two mass scales $\Delta\ln\frac{M_\mathrm{I}}{M_Z}$ and $\Delta\ln\frac{M_\mathrm{U}}{M_Z}$ as shown in Table 4, in terms of superheavy masses. The last term (with the numerical entries) denotes threshold contributions at $\mu = M_Z$. From Table 6, it can be seen that the threshold contributions due to the superheavy masses M_{16} or M_{126} to $\Delta\ln\frac{M_\mathrm{U}}{M_Z}$ are absent for all the models and the threshold contribution due to the representation $\underline{54}$ cancels out from models IV, V and VI. These cancellations are understood by a theorem by Mohapatra [36]. As before the masses are allowed to vary between $\frac{1}{5}$ and 5 times the scale of the relevant symmetry breaking, i.e. M_I or M_U. Numerical solutions to different allowed values of mass scales corresponding to different choices of superheavy masses are presented in Table 7. In Table 7, for models I, IV (II, V) we have the intermediate scale $M_\mathrm{I} \simeq 10^{14}(10^{13})$ GeV with unification scale $M_\mathrm{U} \simeq 10^{16}$ GeV for a reasonable choice of the superheavy masses. In case of models III, VI the intermediate scale can be as low as 10^{10} GeV with unification scale 6.3×10^{15} GeV.

5 Summary and conclusion

While investigating the possibility of G_{2213} intermediate gauge symmetry in SUSY $SO(10)$, we have considered a

Table 6. Threshold effects on the mass scales in different SUSY $SO(10)$ models using superheavy masses

MODEL	Representation content	$\Delta \ln \frac{M_I}{M_Z}$	$\Delta \ln \frac{M_U}{M_Z}$
I	$\underline{210}, \underline{16} \oplus \overline{\underline{16}}, \underline{10}$	$-\frac{1}{2}\ln\frac{M_{16}}{M_U} + \frac{1}{2}\ln\frac{M_{10}}{M_U}$ -1.33	$-\frac{1}{4}\ln\frac{M_{210}}{M_U} - \frac{1}{4}\ln\frac{M_{10}}{M_U}$ $+0.117$
II	$\underline{210}, 2(\underline{16} \oplus \overline{\underline{16}}), \underline{10}$	$\frac{1}{4}\ln\frac{M_{210}}{M_U} - 2\ln\frac{M_{16}}{M_U}$ $+\frac{5}{4}\ln\frac{M_{10}}{M_U} + \ln\frac{M'}{M_I}$ -2.78	$-\frac{1}{4}\ln\frac{M_{210}}{M_U} - \frac{1}{4}\ln\frac{M_{10}}{M_U}$ $+0.122$
III	$\underline{210}, \underline{126} \oplus \overline{\underline{126}}, \underline{10}$	$-\frac{1}{2}\ln\frac{M_{210}}{M_U} + \frac{5}{2}\ln\frac{M_{126}}{M_U}$ $-\ln\frac{M_{10}}{M_U} - 2\ln\frac{M'}{M_I}$ -1.56	$-\frac{1}{4}\ln\frac{M_{210}}{M_U} - \frac{1}{4}\ln\frac{M_{10}}{M_U}$ $+0.105$
IV	$\underline{45}, \underline{54}, \underline{16} \oplus \overline{\underline{16}}, \underline{10}$	$-\frac{1}{2}\ln\frac{M_{16}}{M_U} + \frac{1}{2}\ln\frac{M_{10}}{M_U}$ -1.33	$-\frac{1}{4}\ln\frac{M_{45}}{M_U} - \frac{1}{4}\ln\frac{M_{10}}{M_U}$ $+0.117$
V	$\underline{45}, \underline{54}, 2(\underline{16} \oplus \overline{\underline{16}}), \underline{10}$	$\frac{1}{4}\ln\frac{M_{45}}{M_U} - 2\ln\frac{M_{16}}{M_U}$ $+\frac{5}{4}\ln\frac{M_{10}}{M_U} + \ln\frac{M'}{M_I}$ -2.78	$-\frac{1}{4}\ln\frac{M_{45}}{M_U} - \frac{1}{4}\ln\frac{M_{10}}{M_U}$ $+0.122$
VI	$\underline{45}, \underline{54}, \underline{126} \oplus \overline{\underline{126}}, \underline{10}$	$-\frac{1}{2}\ln\frac{M_{45}}{M_U} - \frac{5}{2}\ln\frac{M_{126}}{M_U}$ $-\ln\frac{M_{10}}{M_U} - 2\ln\frac{M'}{M_I}$ -1.56	$-\frac{1}{4}\ln\frac{M_{45}}{M_U} - \frac{1}{4}\ln\frac{M_{10}}{M_U}$ $+0.105$

Table 7. Predictions on mass scales M_I and M_U including the threshold effect with superheavy masses

Model	Two-loop (GeV)	M'	M_{210} or M_{45}	M_{16}	M_{126}	M_{10}	M_I (GeV)	M_U (GeV)
I,IV	$M_I = 10^{16.9}$	–	$\frac{1}{5}M_U$	$\frac{1}{5}M_U$	–	$\frac{1}{5}M_U$	2.1×10^{16}	3.2×10^{16}
	$M_U = 10^{16.11}$	–	$\frac{1}{5}M_U$	$5M_U$	–	$\frac{1}{5}M_U$	9.7×10^{14}	3.2×10^{16}
II,V	$M_I = 10^{17.6}$	$\frac{1}{5}M_I$	$\frac{1}{5}M_U$	$\frac{1}{5}M_U$	–	$\frac{1}{5}M_U$	1.1×10^{16}	3.3×10^{16}
	$M_U = 10^{16.12}$	$\frac{1}{5}M_I$	$\frac{1}{5}M_U$	$5M_U$	–	$\frac{1}{5}M_U$	1.7×10^{13}	3.3×10^{16}
III,VI	$M_I = 10^{15.2}$	$\frac{1}{5}M_I$	$5M_U$	–	$\frac{1}{5}M_U$	$5M_U$	2.3×10^{10}	6.3×10^{15}
	$M_U = 10^{16.11}$	$\frac{1}{5}M_I$	$5M_U$	–	$\frac{1}{5}M_U$	M_U	1.1×10^{11}	9.5×10^{15}

number of models. At the two-loop level, we have noted that except for the non-minimal models with $3(\underline{16} \oplus \overline{\underline{16}})$, $\underline{210}$, $\underline{10}$ [10] and those of [13,14,16] the RGEs in the minimal models do not permit the G_{2213} intermediate breaking scale M_I to be substantially lower than M_U when both gravitational and threshold effects are ignored. Including gravitational corrections in the minimal model, we observe that the prediction on the unification mass remains unaffected by such Planck-scale-induced gravitational effects whereas the intermediate scale can be lowered by at most two orders of magnitude through such corrections.

Including threshold corrections, we have considered various minimal and non-minimal models and obtained $M_I \simeq 10^{10}$–10^{13} GeV with high unification scale using plausible values of the effective mass parameters but without using an additional number of Higgs scalars at M_I (models I, III, IV and VI with $n_{126} = 1$ or $n_{16} = 1$). Our choices of the effective mass parameters are similar to those used in earlier investigations [22,29] and the choices of superheavy masses near the thresholds are similar to those used in earlier analyses [30–34]. An important feature of the analytic result is that the GUT scale is unaffected by the spreading of masses near the intermediate scale although the intermediate scale is itself changed significantly by the superheavy masses near the GUT scale. Thus the proton lifetime predictions in the model for the $p \to e + \pi^0$ mode are unchanged by gravitational or intermediate-scale threshold corrections. Another important aspect of this analysis is that even if the spreading of the masses near the two thresholds are only a few times heavier or lighter than the corresponding scales, the models result in $M_\mathrm{I} \simeq 10^{11}$–$10^{13}$ GeV either with effective mass parameters or with superheavy masses. We further observe that low-mass right-handed gauge bosons in the range 1–10 TeV are permitted in the model only when threshold effects are computed with effective mass parameters. All relevant superheavy masses contributing to threshold effects have been assumed to be bare masses as their wave function renormalization has been shown to be cancelled out by two-loop effects [35].

It may be noted that while the use of $\underline{126} \oplus \overline{\underline{126}}$ permits the implementation of the conventional see-saw mechanism for neutrino masses with R-parity conservation, it is possible to use a generalized mechanism [37,38] with the choice $\underline{16} \oplus \overline{\underline{16}}$, such that one can get a see-saw-like formula for light neutrino masses. In the latter case R-parity is violated and one needs to impose additional discrete symmetries to maintain the stability of the proton. We thus conclude that $G_{2213}(g_{2\mathrm{L}} \neq g_{2\mathrm{R}})$ with minimal choice of Higgs scalars is allowed as an intermediate gauge symmetry in the SUSY $SO(10)$ model. The right-handed Majorana neutrinos associated with the intermediate scales obtained in this analysis are compatible with the observed indications for light Majorana neutrino masses through the see-saw mechanism [5,8,9].

Recently interesting investigations have been made in SUSY left–right gauge models while embedding G_{2213} in SUSY $SO(10)$ as an intermediate gauge group in the presence of higher-dimensional operators [39]. It would be even more interesting to study the impact of threshold effects in such models where certain light degrees of freedom are naturally allowed and R-parity conservation is guaranteed.

Acknowledgements. The work of M.K.P and C.R.D. is supported by the project No. SP/S2/K-30/98 of Department of Science and Technology, Govt. of India.

References

1. P. Langacker, M. Luo, Phys. Rev. D **44**, 817 (1991)
2. J. Ellis, S. Kelley, D.V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); U. Amaldi, W. de Boer, H. Furstanau, Phys. Lett. B **260**, 447 (1991); F. Anselmo, L. Cifarelli, A. Peterman, A. Zichichi, Nuovo Cimento A **104**, 1827 (1991)
3. I. Antoniadis, J. Ellis, S. Kelley, D.V. Nanopoulos, Phys. Lett. B **272**, 31 (1991); T.G. Rizzo, Phys. Rev. D **45**, 3903 (1992); U. Amaldi, W. de Boer, P.H. Frampton, H. Furstanau, J.T. Liu, Phys. Lett. B **281**, 374 (1992)
4. J. Ellis, S. Kelley, D.V. Nanopoulos, Nucl. Phys. B **373**, 55 (1992); Phys. Lett. B **287**, 95 (1992); T. Blazek, M. Carena, S. Raby, C.E.M. Wagner, Phys. Rev. D **56**, 6919 (1997); M. Carena, J. Ellis, S. Lola, C.E.M. Wagner, Eur. Phys. J. C **12**, 507 (2000)
5. M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity, Proceedings of the Workshop, Stony Brook, New York, 1997, edited by P.van Niewenhuizen, D.Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in Proceedings of the Workshop on Unified Theory and baryon Number in the Universe, T. Sukuba, Japan, 1997, edited by O. Sawada, A. Sugimoto (KEK, Japan, 1979); R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981)
6. R.N. Mohapatra, J.C. Pati, Phys. Rev. D **11**, 566 (1975); R.N. Mohapatra, G. Senjanovic, Phys. Rev. D **12**, 1502 (1975)
7. J.C. Pati, A. Salam, Phys. Rev. Lett. **31**, 661 (1973); Phys. Rev. D **10**, 275 (1974)
8. Superkamiokande Collaboration, Y. Fukuda et al. Phys. Rev. Lett. **82**, 2644 (1999); T. Kajita, in Proceedings of PASCOS 99 (1999)
9. H. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney, I.V. Krivosheina, Mod. Phys. Lett. A **16**, 2409 (2001)
10. F. Vissani, A.Yu. Smirnov, Phys. Lett. B **341**, 173 (1994); A. Brignole, H. Murayama, R. Rattazi, LBL-35774 (1994)
11. G.K. Leontaris, S. Lola, G.G. Ross, Nucl. Phys. **454**, 25 (1995); M.K. Parida, N.N. Singh, Phys. Rev. D **59**, 032005 (1999)
12. N.G. Deshpande, E. Keith, T.G. Rizzo, Phys. Rev. Lett. **70**, 3189 (1993)
13. M. Bando, J. Sato, T. Takahashi, Phys. Rev. D **52**, 3076 (1995)
14. Dae-Gyu Lee, R.N. Mohapatra, Phys. Rev. D **52**, 4125 (1995)
15. B. Brahmachari, R.N. Mohapatra, Phys. Lett. B **357**, 566 (1995)
16. E. Ma, Phys. Lett. B **344**, 164 (1995); Phys. Rev. D **51**, 236 (1995)
17. M.K. Parida, Proceedings of the third Workshop on High Energy Physics, 1994, Pramana (Journal of Physics) Suppl. **45**, 209 (1995)
18. M.K. Parida, P.K. Patra, Phys. Rev. Lett. **66**, 858 (1991); M.K. Parida, P.K. Patra, Phys. Rev. Lett. **68**, 754 (1992)
19. M.K. Parida, Phys. Rev. D **57**, 2736 (1998)
20. N.G. Deshpande, B. Dutta, E. Keith, Phys. Lett. B **384**, 116 (1996)
21. Q. Shafi, C. Witterich, Phys. Rev. Lett. **51**, 875 (1984); C.T. Hill, Phys. Lett. B **135**, 47 (1984)
22. P. Langacker, N. Polonsky, Phys. Rev. D **47**, 4028 (1993)
23. L. Hall, U. Sarid, Phys. Rev. Lett. **70**, 26 (1993)
24. M.K. Parida, P.K. Patra, Phys. Rev. D **39**, 2000 (1989); M.K. Parida, P.K. Patra, Phys. Rev. D **44**, 2179 (1991); M.K. Parida, P.K. Patra, A.K. Mohanty, Phys. Rev. D **39**, 316 (1989)
25. M.K. Parida, P.K. Patra, Phys. Lett. B **324**, 45 (1990)
26. D. Chang, R.N. Mohapatra, M.K. Parida, Phys. Rev. Lett. **52**, 1072 (1984); Phys. Rev. D **30**, 1052 (1984)
27. Particle Data Group, L. Montanet et al., Phys. Rev. D **50**, 1173 (1994); R.M. Barnett et al., Phys. Rev. D **54**, 1 (1996); G. Altarelli, hep-ph/9719434
28. D.R.T. Jones, Phys. Rev. D **25**, 581 (1982)

29. M. Carena, S. Pokorski, C.E.M. Wagner, Nucl. Phys. B **406**, 59 (1993)
30. S. Weinberg, Phys. Lett. B **91**, 51 (1980); L. Hall, Nucl. Phys. B **178**, 75 (1981); B. Ovrut, H. Schnitzer, Nucl. Phys. B **179**, 75 (1981); W.J. Marciano, in Field Theory in Elementary Particles, edited by A. Perlmutter (Plenum, New York 1982), Proceedings of Orbis Scientiae Vol. 19; in Fourth workshop on Grand Unification, Philadelphia, Pennsylvania, 1983, edited by H.A. Weldon, P. Langacker, P.J. Steinhardt (Birkhauser, Boston 1983)
31. M.K. Parida, Phys. Lett. B **196**, 163 (1987); M.K. Parida, C.C. Hazra, Phys. Rev. D **40**, 3047 (1989)
32. R.N. Mohapatra, M.K. Parida, Phys. Rev. D **47**, 264 (1993); Dae-Gyu Lee, R.N. Mohapatra, M.K. Parida, M. Rani, Phys. Rev. D **51**, 229 (1995); M. Rani, M.K. Parida, Phys. Rev. D **49**, 3704 (1994); M.L. Kynshi, M.K. Parida, Phys. Rev. D **49**, 3711 (1994)
33. R. Barbieri, L. Hall, Phys. Rev. Lett. **68**, 752 (1992); J. Hisano, H. Murayama, T. Yanagida, Nucl. Phys. B **402**, 46 (1993); M.B. Einhorn, D.R.T. Jones, Nucl. Phys. B **196**, 475 (1982)
34. M.K. Parida, Phys. Rev. D **58**, 11006 (1998)
35. M. Shifman, Int. J. Mod. Phys. A **11**, 5761 (1996)
36. R.N. Mohapatra, Phys. Lett. B **285**, 235 (1992)
37. R.N. Mohapatra, Phys. Rev. Lett. **47**, 561 (1993); R.N. Mohapatra, J.W.F. Valle, Phys. Rev. D **34**, 1642 (1986)
38. Dae-Gyu Lee, R.N. Mohapatra, Phys. Rev. D **52**, 4125 (1995); Z. Chacko, R.N. Mohapatra, Phys. Rev. D **58**, 015003 (1998)
39. C.S. Aulakh, A. Melfo, A. Rasin, G. Senjanovic, Phys. Rev. D **58**, 115007 (1998); C.S. Aulakh, A. Melfo, G. Senjanovic, Phys. Rev. D **57**, 4174 (1998); C.S. Aulakh, B. Bajc, A. Melfo, A. Rasin, G. Senjanovic, Phys. Lett. B **460**, 325 (1999)